A Denotational Study of Mobility

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Introduction

Two Main Points of View on Modelling Processes

- Operational POV (π-calculus. . .)
  - Low level, double-edged: easy mobility but difficult to abstract
  - unsettled theory so many variants
  - issues with compositionality: bound prefixes and guards
  - denotations exist but not practical
- Denotational POV (CSP)
  - denotational (tr, fail, div) and compositional by design
  - supports refinement
  - but no easy way to account for mobility

Our Approach: Mobility in a Denotational Way

- Heavily inspired by CSP but integrated model (decorated traces)
- π-like mobility but compositional $\Longrightarrow$ fully denotational model
- Support for refinement

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Outline

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Representing Behaviours

The problem

- Full representation of behaviour? branching structure (LTS)
- Set of process traces: information lost
- Traces + failures,divergences: hard to introduce mobility

What we wanted

- Traces but with as much information as the LTS
- How: link observations to *where* and *when* in LTS
  $\Longrightarrow$ *locations* !
- LTS can be rebuilt from decorated traces: no information lost

Basic Example

The basics

Observation (::location): input *channel*? output *channel*!*value*, or ✓

Location: origin: ϵ, next: $\triangleright$ and choice: $\diamond^{number\ of\ branches}_{branch\ number}$, weak variants $\tilde{\triangleright}$ and $\tilde{\diamond}^{j}_{i}$

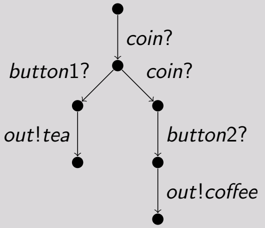
Process (*not* mobile)

$coin?.(button1?.out!tea + coin?.button2?.out!coffee)$

What does this process do?

Behaviour

LTS



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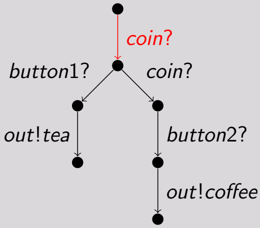

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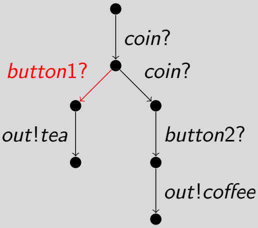

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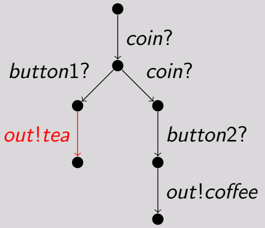

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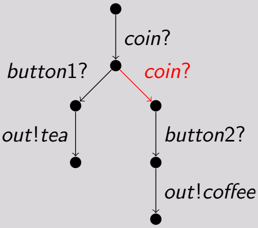

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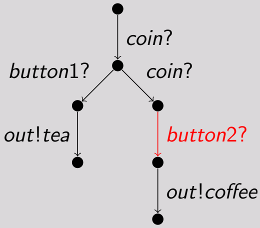

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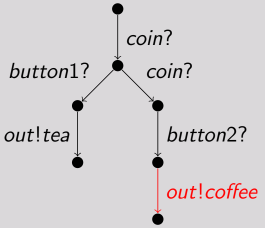

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Which locations why? What is an absolute location?

Behaviour

LTS

$$\begin{tikzcd}
 & \epsilon \arrow[d, "coin?"] & \\
 & \triangleright \arrow[dl, "button1?"'] \arrow[dr, "coin?"] & \\
\diamond^2_1 \arrow[d, "out!tea"'] & & \diamond^2_2 \arrow[d, "button2?"] \\
\triangleright & & \triangleright \arrow[d, "out!coffee"] \\
 & & \triangleright
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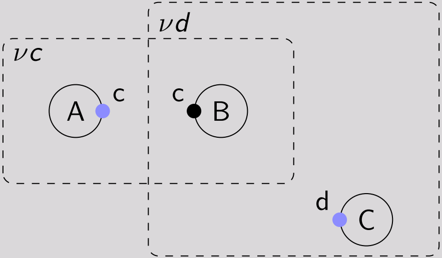
Outline

About Mobility

Physical vs Logical Mobility

A process is mobile if it changes neighbours

How Can a Process Change Neighbours?



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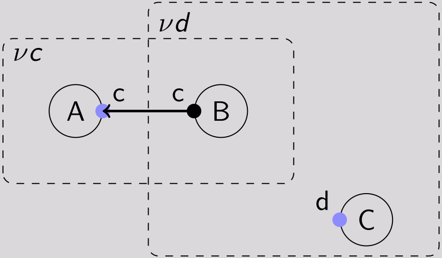

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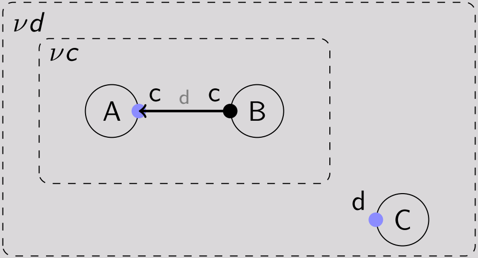

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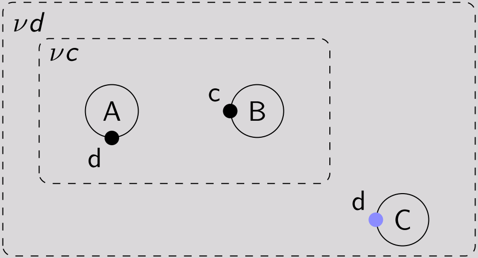

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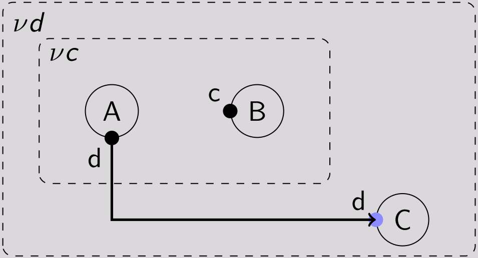


The Modeling Problems

Two main problems

- Binders
- Guards

Binders in mobile languages

- Binders: dynamic names (escaped names and inputs)
- π-calculus operational, mixes free and bound names
- Solution: binders are uniquely identified by when/where created
- advantage: fresh by construction, avoid α-conversion issues

Guards

- Reminder: $[\varphi]P$ means if φ then P
- Not observations, but necessary for compos. Where do they go?
- Solution: in locations

Example 2

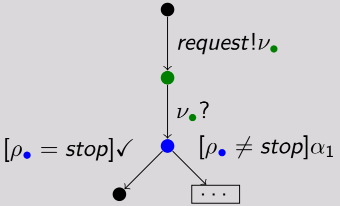
Process with guards and extrusion

$$(\nu\ in)request!in.in?out.$$
$$([out = stop]SKIP + [out \neq stop]Communicate(in, out))$$

What does this process do?

Behaviour

LTS



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Extruded names: ν_{where}

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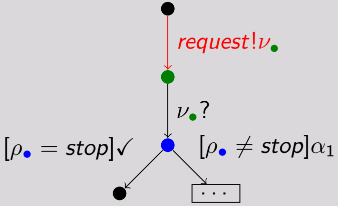

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Input *observations* have no object; received names: ρ_{where}

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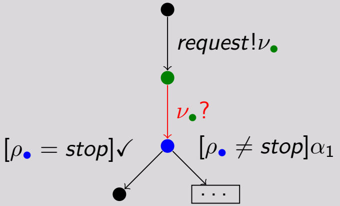

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In traces the guard of an observation prefixes its location

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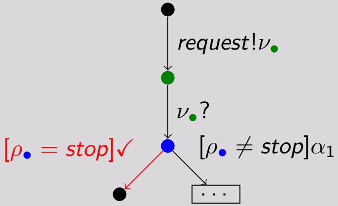

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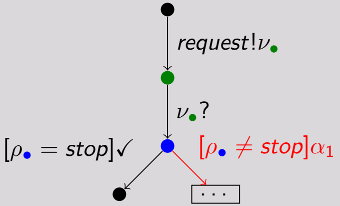

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Outline

1. Basics: Locations
2. Mobility
3. Equivalence & Refinement

Equivalence and Normal Forms

Dealing with redundancy

- Problem: model very fine-grained
- Solution: rewrite rules to trim redundancy

Theorem

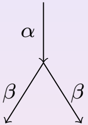
Let T be a trace set. Suppose T_1 and T_2 such that $T \longrightarrow^* T_1 \not\rightarrow$ and $T \longrightarrow^* T_2 \not\rightarrow$. Then $T_1 = T_2 = \widehat{T}$.

Interest

- Equivalence checking: normalise then test isomorphism
- Much simpler than existing equivalence checking for mobility
- Only possible because no binders in semantic

Example

$$P = \alpha.(\beta + \beta)$$
$$traces(P) = \{\langle \alpha{::}\triangleright, \beta{::}\diamond^2_1\rangle, \langle \alpha{::}\triangleright, \beta{::}\diamond^2_2\rangle\}$$



$$Q = \alpha.\beta + [a = x \wedge a \neq x]\gamma$$
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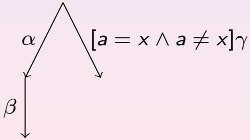

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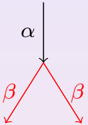

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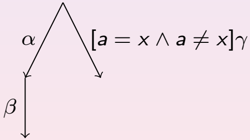

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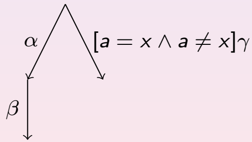

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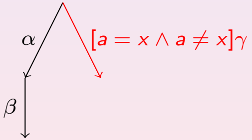

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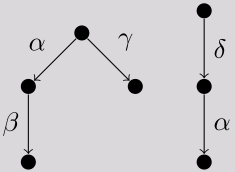
Delayed Sum

What is the delayed sum?

- The way to refinement
- Strict generalisation of process sum
- Grafting any behaviour anywhere in branching structure
- Two parameters: a location and a substitution from symbols to special names

Delayed Sum Example

$$P \overset{\text{def}}{=} \alpha.\beta + \gamma \qquad Q \overset{\text{def}}{=} \delta.\alpha \qquad P +^{Id}_{\epsilon\diamond^{2}_{1}} Q = \alpha.(\beta + \delta.\alpha) + \gamma$$



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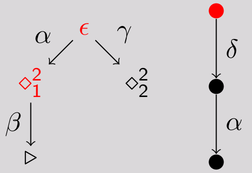

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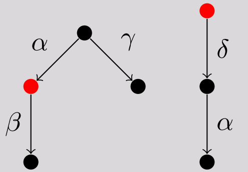
$$P \stackrel{\text{def}}{=} \alpha \bullet \beta + \gamma \qquad Q \stackrel{\text{def}}{=} \delta.\alpha \qquad P +^{Id}_{\epsilon\diamond_1^2} Q = \alpha.(\beta + \delta.\alpha) + \gamma$$



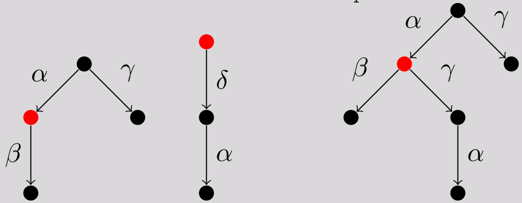
Delayed Sum

What is the delayed sum?

- The way to refinement
- Strict generalisation of process sum
- Grafting any behaviour anywhere in branching structure
- Two parameters: a location and a substitution from symbols to special names

Delayed Sum Example

$$P \overset{\mathrm{def}}{=} \alpha.\beta + \gamma \qquad Q \overset{\mathrm{def}}{=} \delta.\alpha \qquad P +^{Id}_{\epsilon\diamond^2_1} Q = \alpha.(\beta + \delta.\alpha) + \gamma$$



Refinement

Definition

$P \sqsubseteq Q \iff \exists \mathcal{R}\mathcal{L} = \bigcup_{i=1}^{n} \{(R_i, l_i, \sigma_i)\}$ s. t.
$P =_\diamond Q +_{l_1}^{\sigma_1} R_1 \ldots +_{l_n}^{\sigma_n} R_n$

Why?

- Refinement relation nearly for free
- The delayed sum cannot be compositional... is refinement?

Conclusion

What we did

- CSP vs π-calculus: a step towards bridging the gap
- Denotational theory for mobility with intuitive refinement
- Operational semantics w/o π-calculus pitfalls
- Axiomatic semantics
- A Hoare-like logic [LAM09]

What next?

- Finish writing the thesis...
- Proving that refinement is compositional
- Equivalence/refinement checking algorithm