On Congruence Property of Scope Equivalence for Concurrent Programs with Higher-Order Communication

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A Formal Model of Concurrent Systems

- the model presented here is
  - a translation of
    - asynchronous local highr-order π-calculus (Sangiorge)
    - into <u>graph rewriting</u>

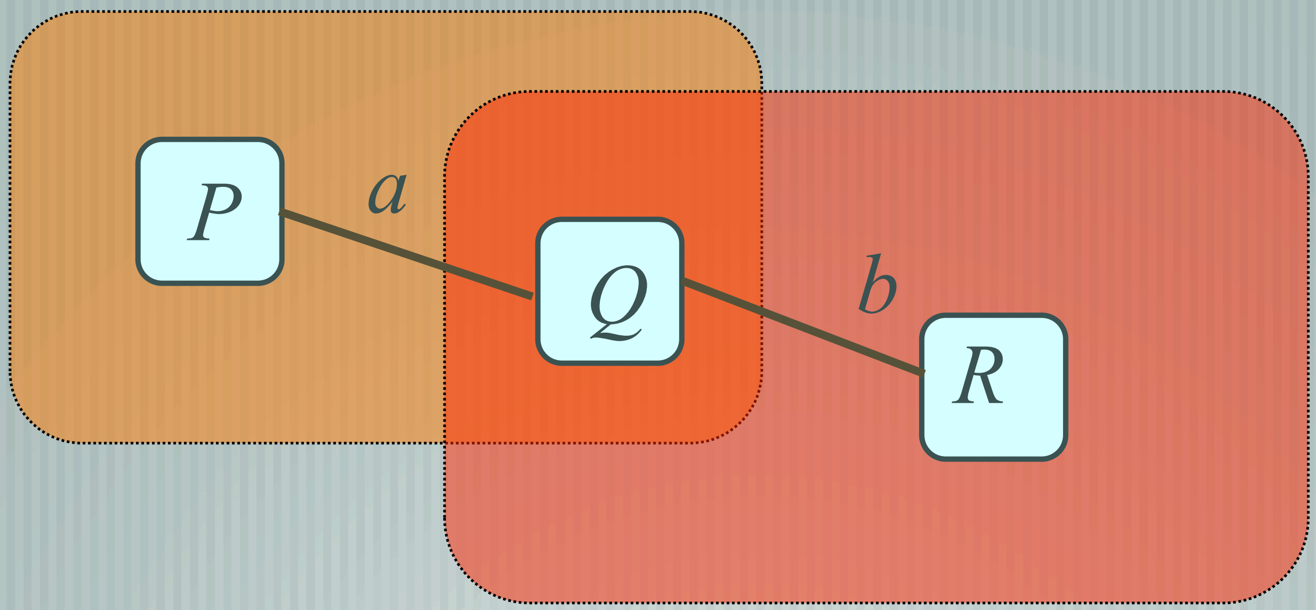
Motivation

- To represent the scopes of channel names precisely
  - ν-operator

$$\nu a(P \mid \nu b(Q \mid R))$$

- Not convenient to express scopes of names for some purpose..

Scopes not nested



- Impossible to represent with a ν-operator

$$\nu a(P \mid \nu b(Q \mid R))$$

We can not decide..

$\vdash \nu a(P \mid \nu b(Q \mid R))$ means......

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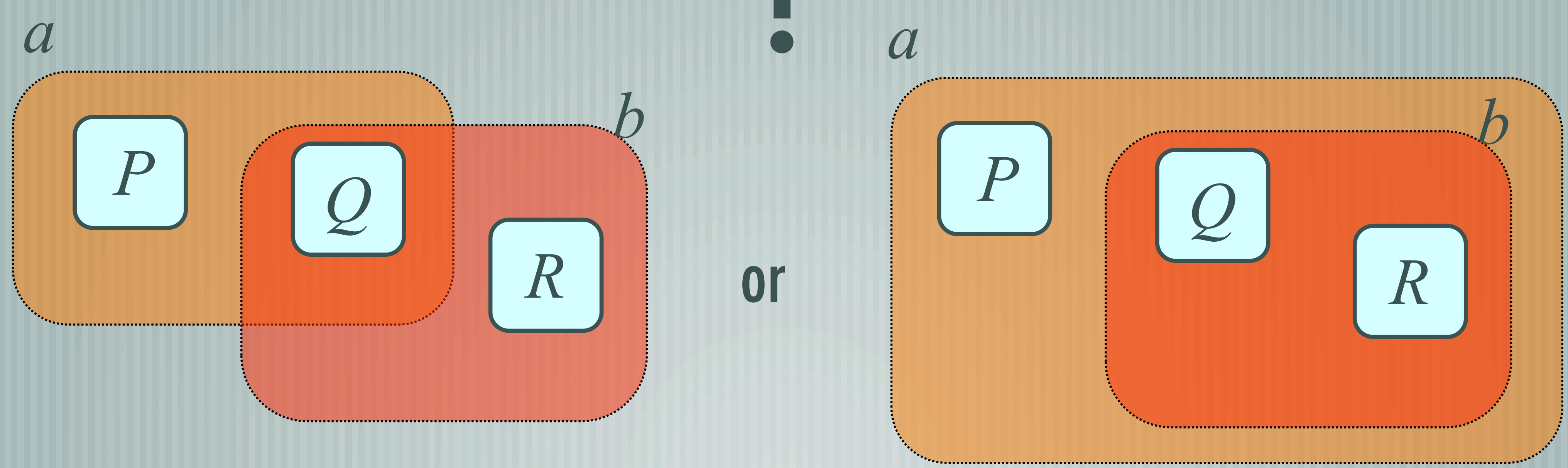
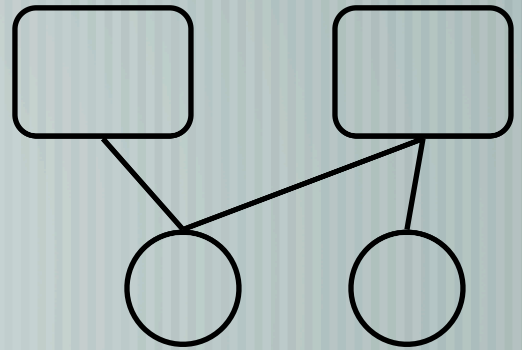


Our approach..

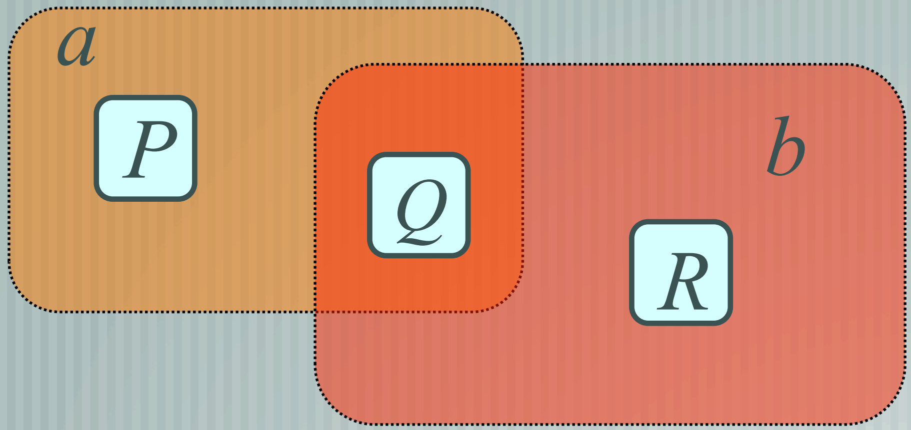
- Our model is based on graph rewriting.
- not based on process algebra.
- a translation of asynchronous higher-order π-calculus into graph rewriting

Basic Idea

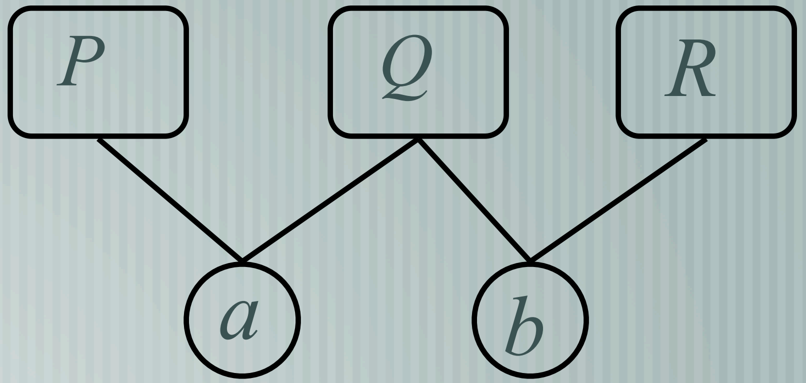
- A system is a collection of *processes* sharing *names*
- A system is represented as a bipartite graph
  - Source nodes ==> processes
  - Sink nodes ==> names
  - There is an edge iff the source nodes is in the scope of the sink node



Basic Idea



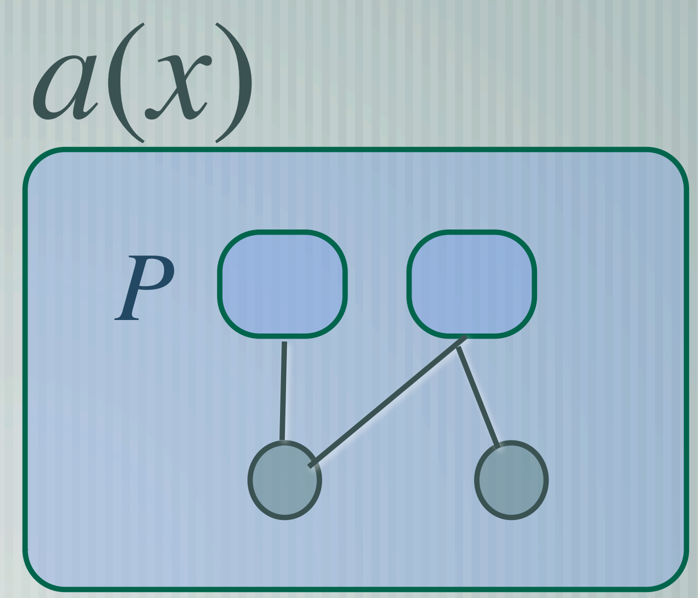
bipartite graph



Processes

- A source node consists of labels for its prefix and its continuation
- Reduce a process by “peeling” the node.

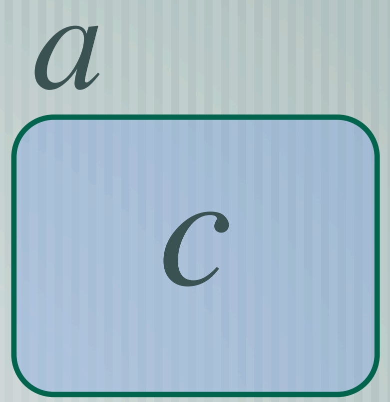
$$a(x).\mathbf{P}$$



Message node

- a message node is a tuple of its subject and its object

$$a<c>$$

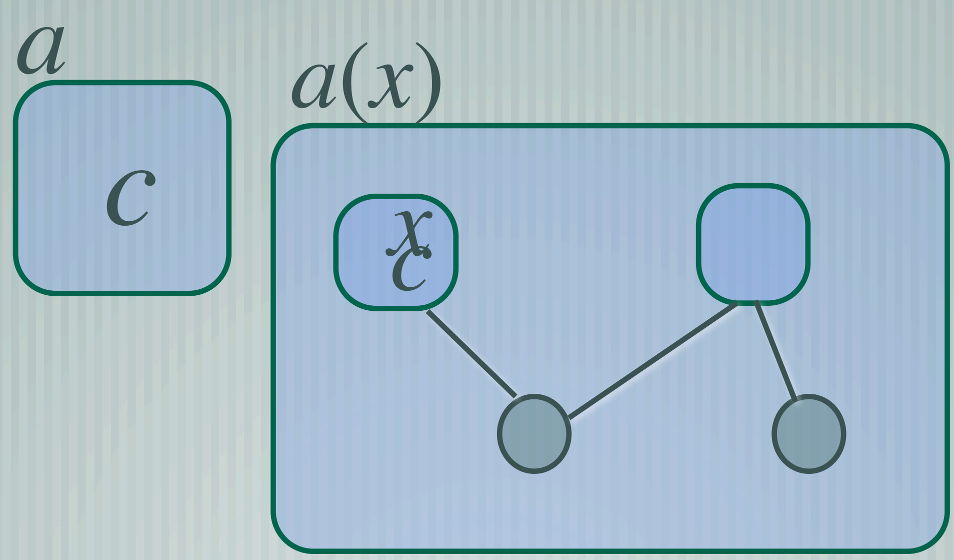


Operational Semantics

- a set of graph rewriting rules
- by translating the rules for the labeled transition system of asynchronous π-calculus into rules for graph rewriting

Rules for graph rewriting

- The rule for message receiving..



Rules for graph rewriting

- If the imported name is new to the receiver, new edges are created

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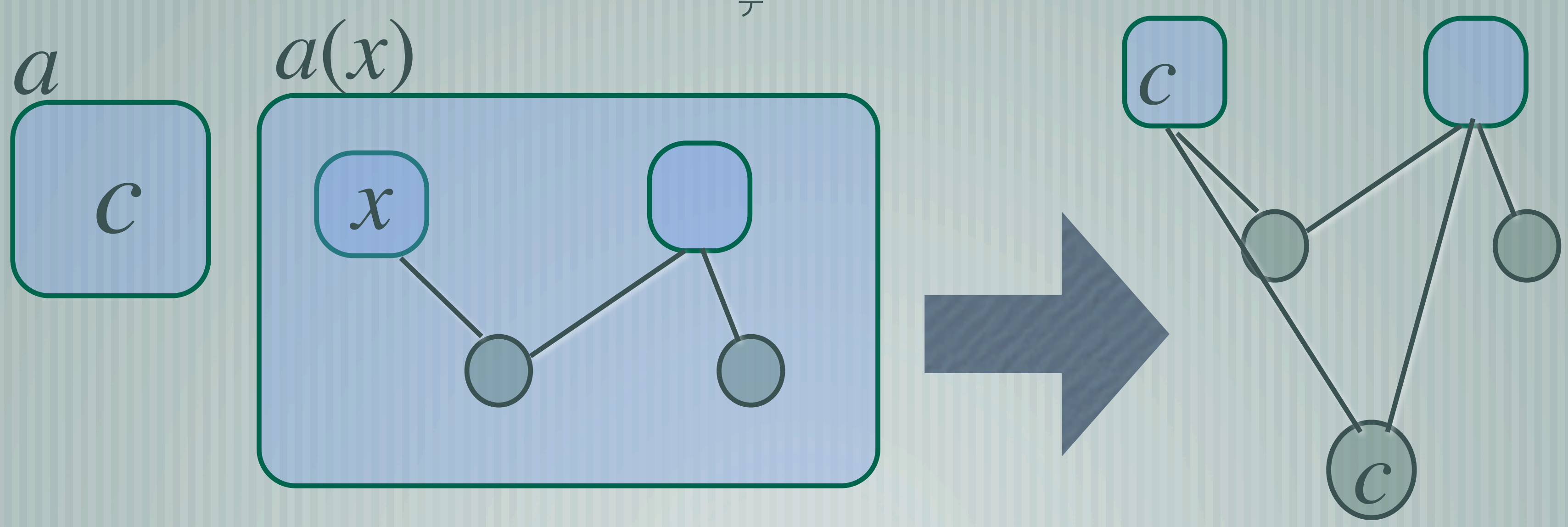
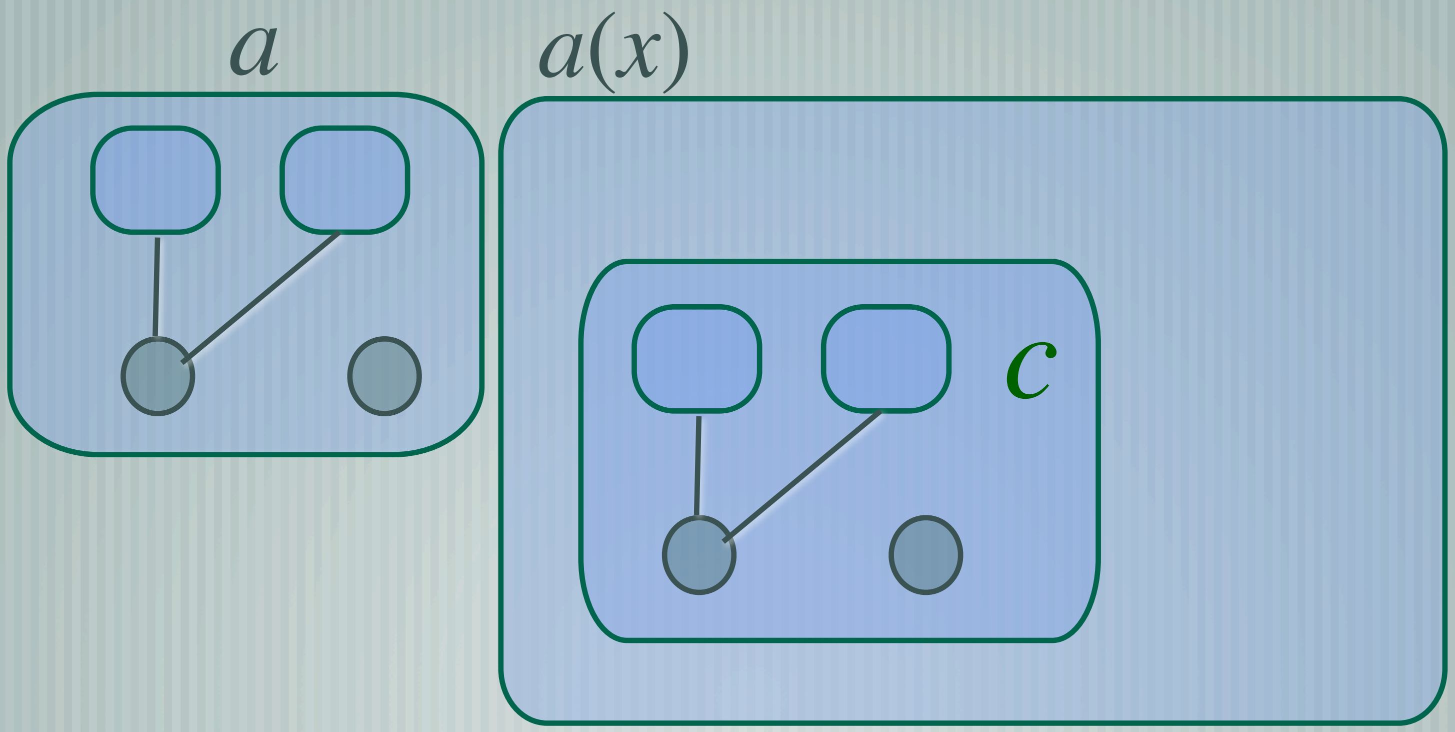


Higher-Order Communication



Scope Equivalence

- We define a new equivalence relation
- to distinguish two processes
  - which are equivalent on their behavior
  - but not for their scopes of names

Example

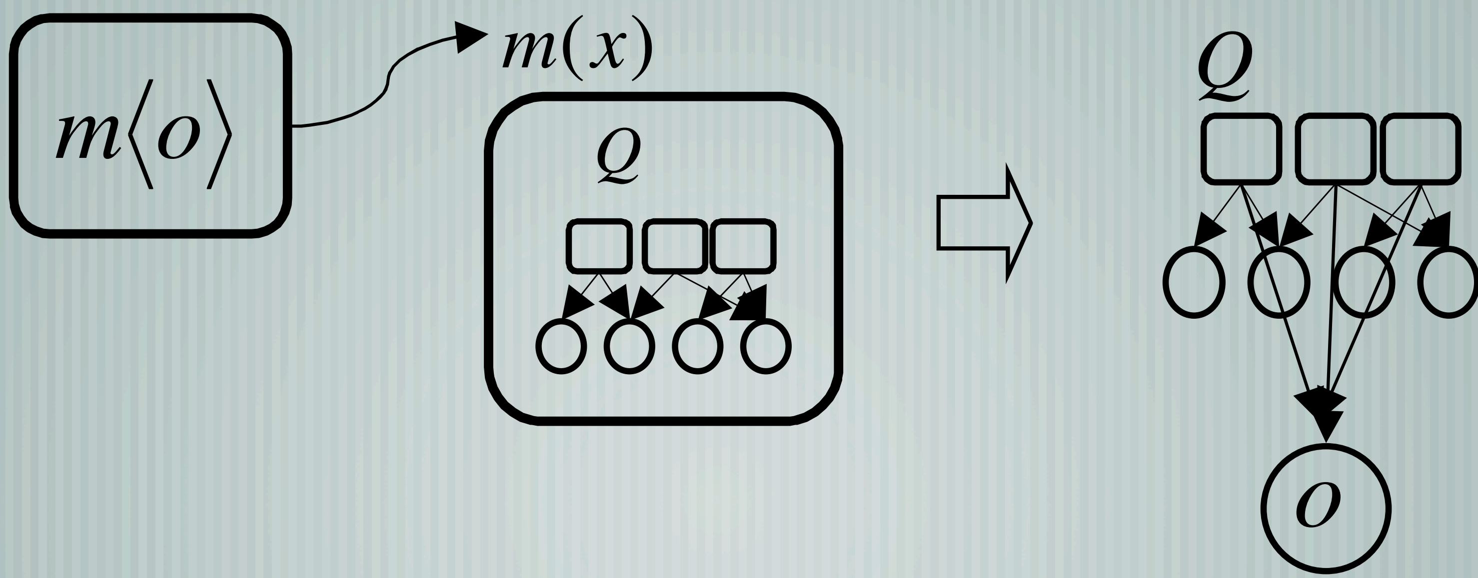
- When x does not occur in Q
  - P_1 and P_2 are equivalent in their behavior
  - but <u>not equivalent</u> for scopes of names
  - $P_1 = m(x).\tau.Q$
  - $P_2 = \nu n(m(u).\ (n<a> \mid n(x).\ Q))$

Example

- Note that Q may be just a specification of the behavior. It does not represent the implementation.
- "x does not occur in Q" does not mean "the imported name no longer exists in Q"
- $P_1 = m(x).\tau.Q$
- If the name receive by $m(x)$ is a secret data which should not be leaked to Q, this P_1 is no good (but P_2 is OK).

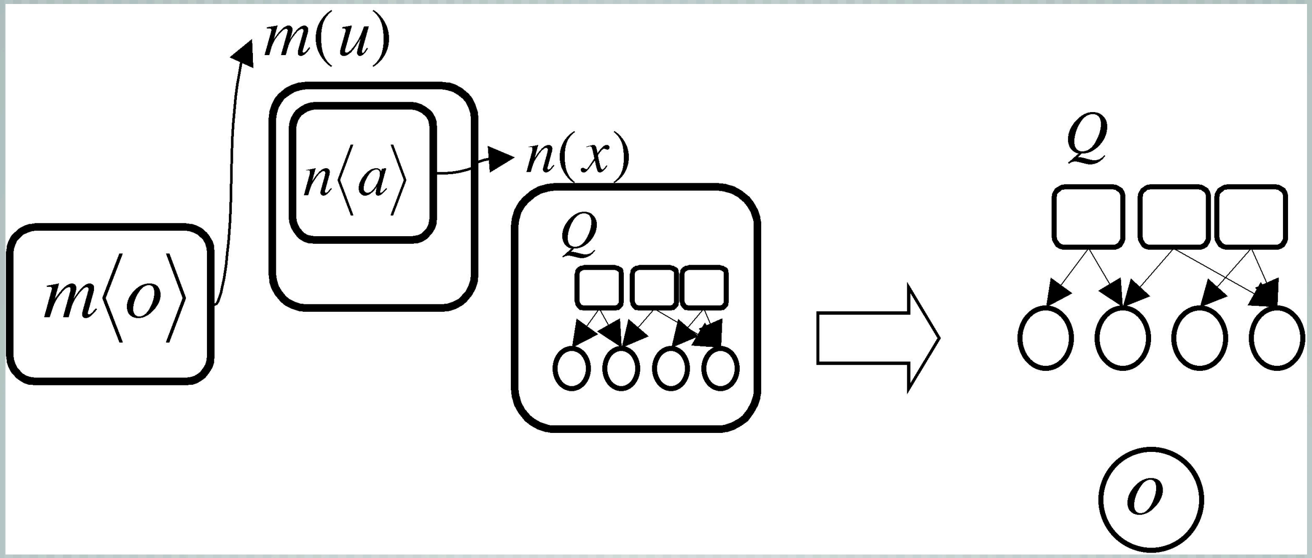
Example

- Behavior equivalences can not tell you the difference.
- The graph rewriting model can represent the difference.



Example

$$P_2 \equiv \nu n(m(u).\ (n\langle a\rangle \mid n\,(x).\ Q))$$



Scope Equivalence

- Define a new equivalence relation that is called scope equivalence that can distinguish these two processes.

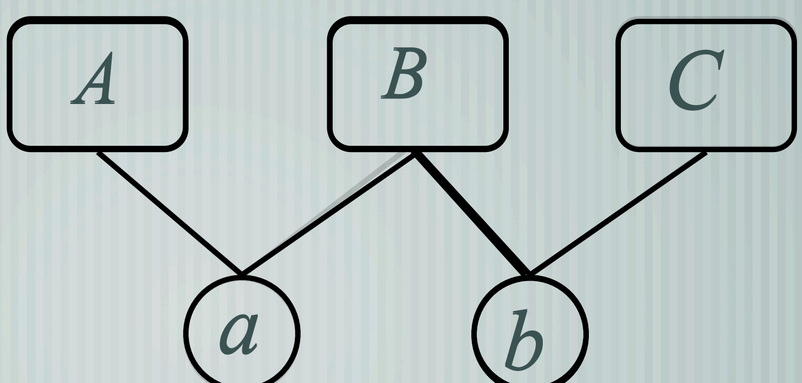
$$P_1 = m(x).\tau.Q$$

$$P_2 = \nu n(m(u).\ (n<a> \mid n(x).\ Q))$$

Definitions

- For a graph P and a name n, P/n is a subgraph of P which consists of
  - source nodes in the scope of n
  - and sink nodes other than n

P/a



Scope Bisimulation

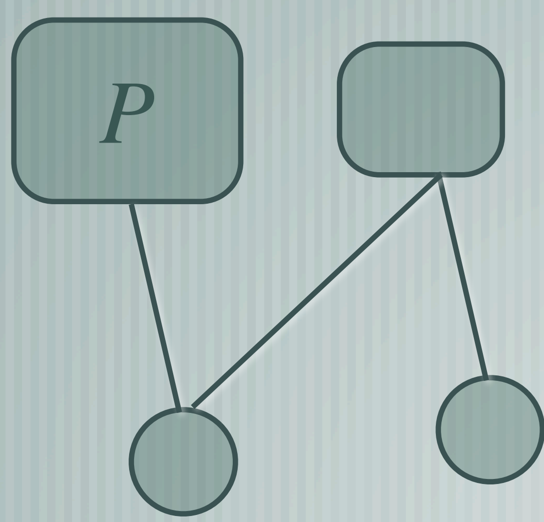
- a relation R is a scope bismulaiton if for any P and Q such that (P, Q) in R,
  - P is an empty graph iff Q is an empty graph
  - the set of source nodes of P/n is empty iff the source nodes Q/n is also empty for any common name n
  - P/n and Q/n are strongly bisimular for any common name n
  - R is a strong bisimulation

Scope Equivalence

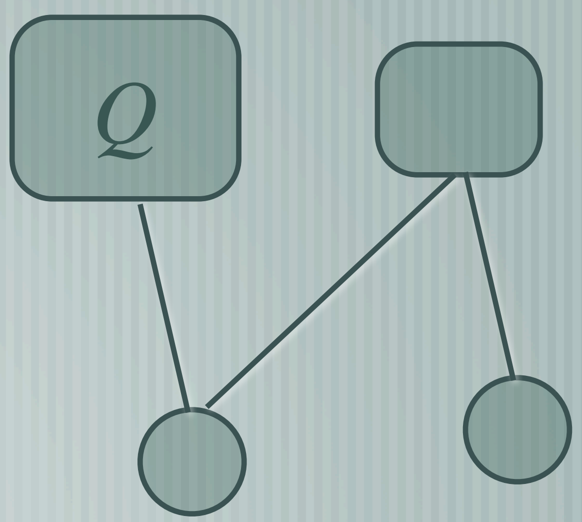
- There exists the largest scope bisimulation
  - which is a equivalence relation
  - congruent w.r.t. contexts (composition, prefix, replication, new name...) in first-order case (ICTAC 08)

Congruence : for higher-order model

- When P and Q are scope equivalent..



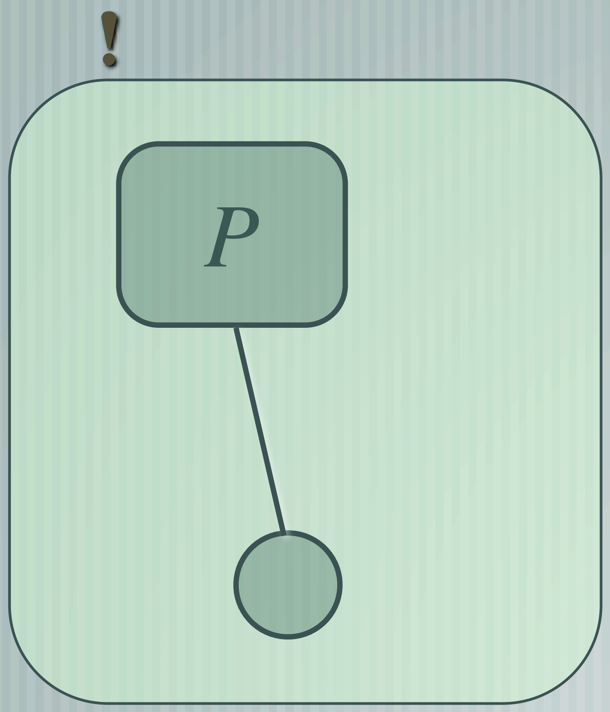
and



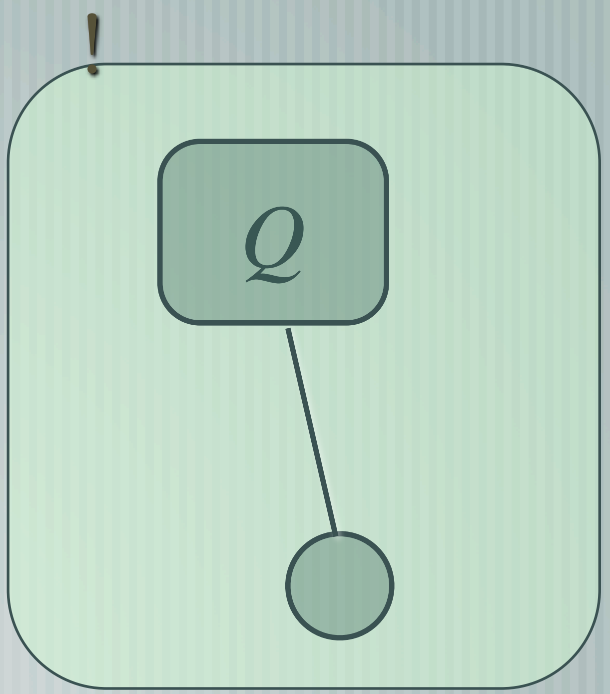
are also equivalent

Congruence(2)

- When P and Q are scope equivalent..



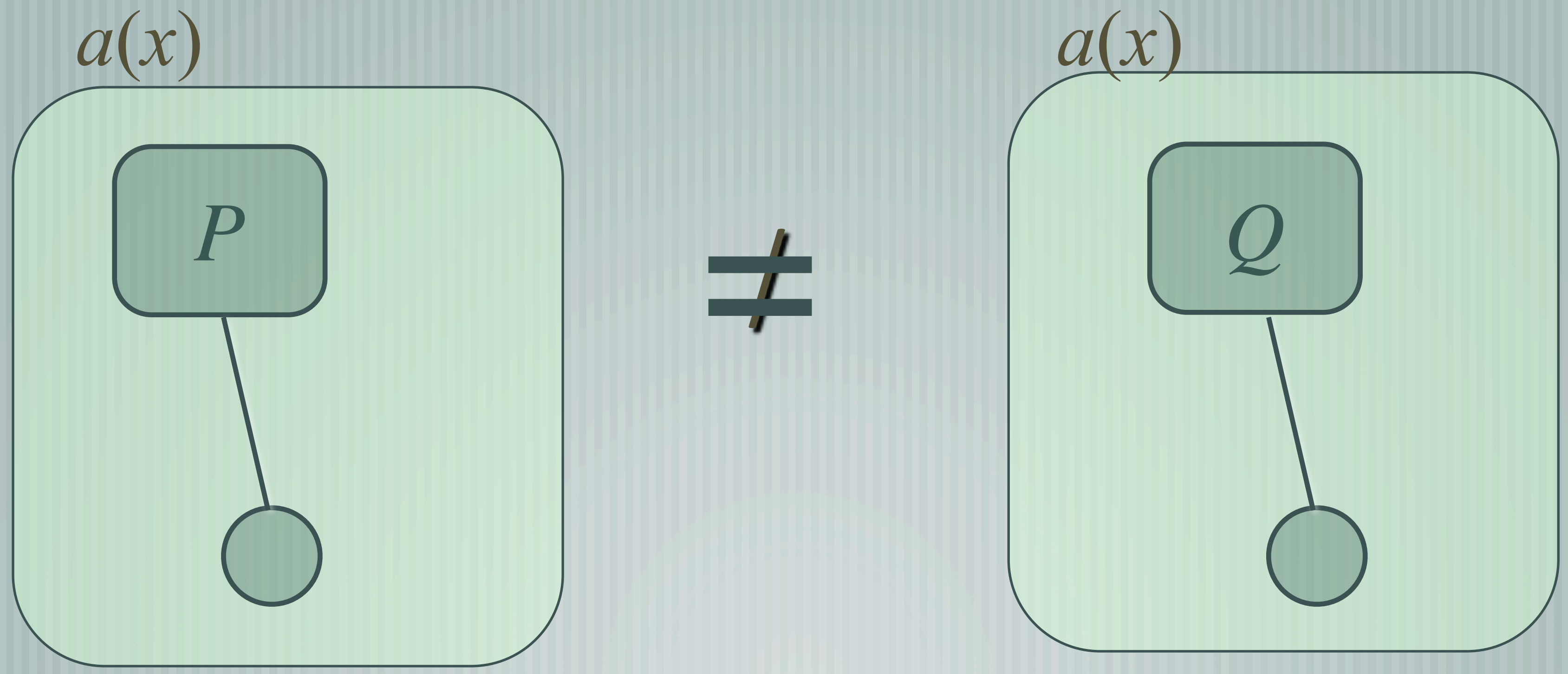
and



are also equivalent

Non Congruence w.r.t. input prefix

P and Q are scope equivalent but....

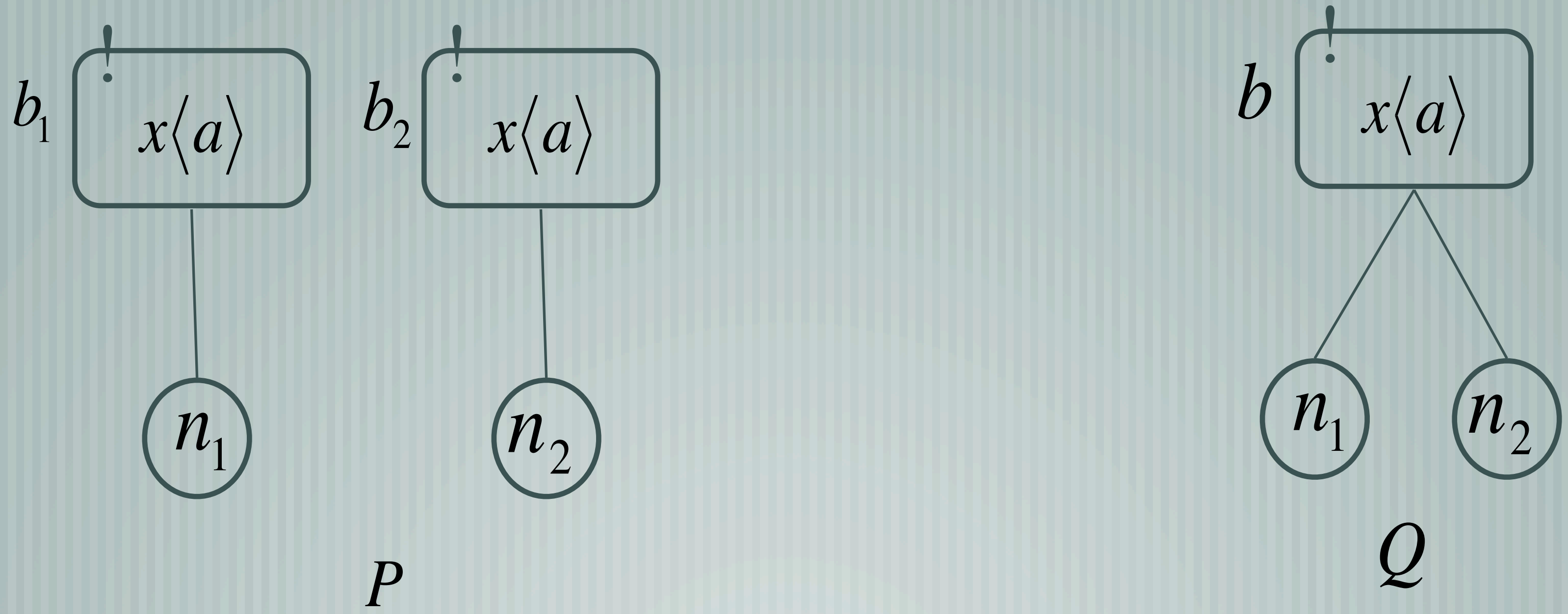


The Non Congruence result

- It comes from....
  - Scope equivalence is NOT congruent w.r.t. higher-order substitution.

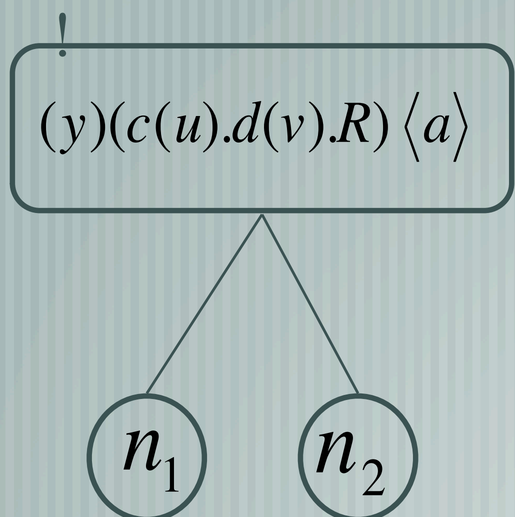
The Counter Example

- P and Q are equivalent.

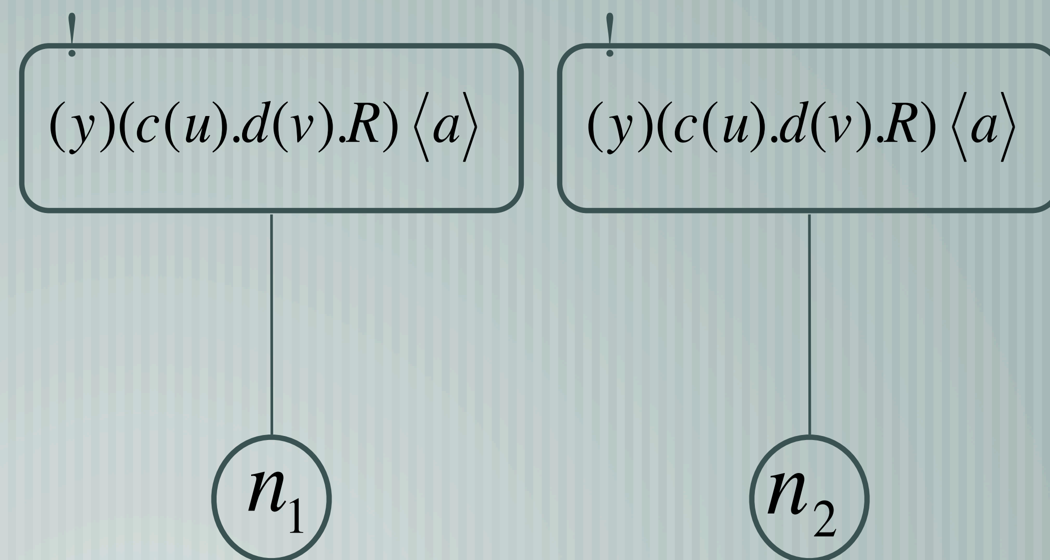


The Counter Example

- Not equivalent after the higher-order substitution.

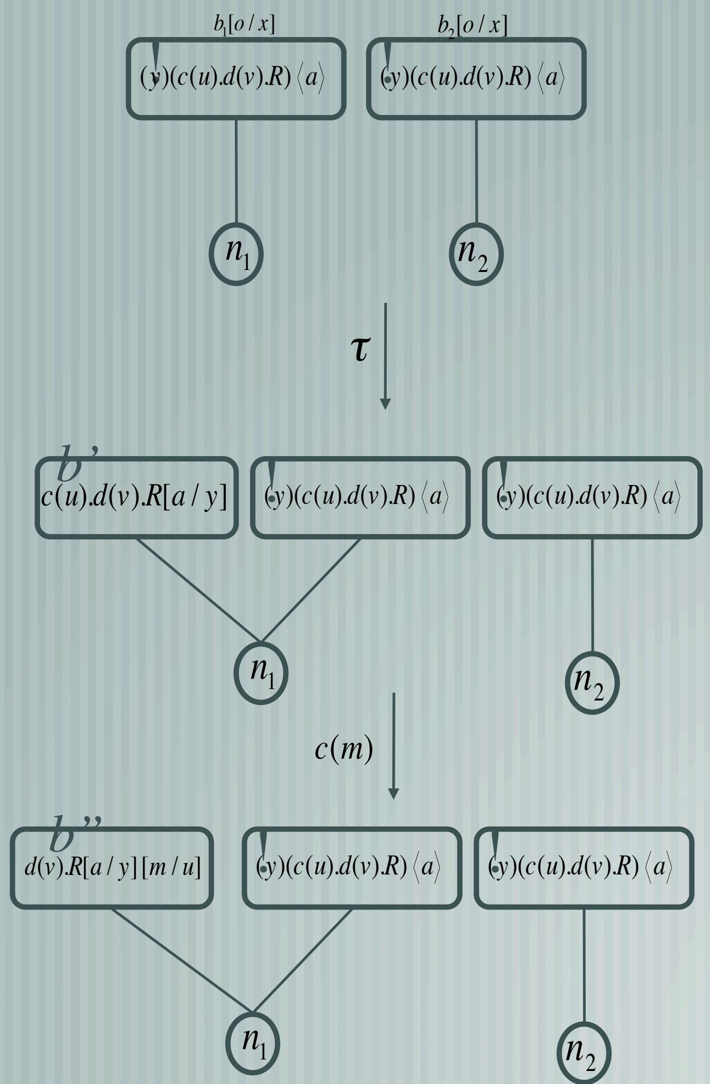


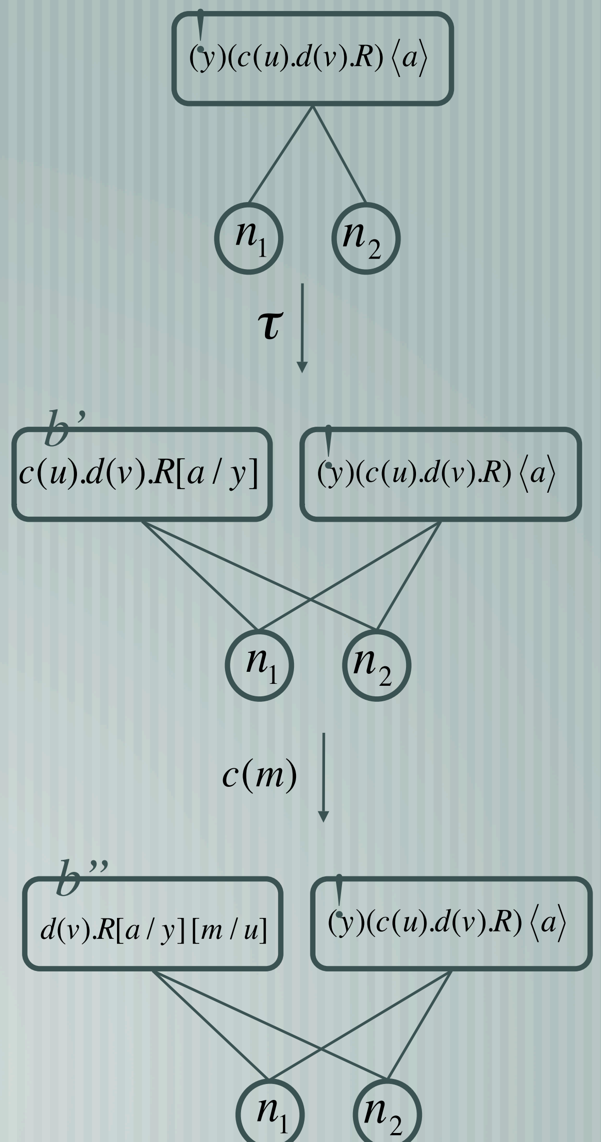
$P[(y)(c(u).d(v).R)\,/\,x]$



$Q[(y)(c(u).d(v).R)\,/\,x]$

The counter example





Conclusion

- A graph rewriting model of concurrent/ distributed systems with higher-order message
- represents scopes of names precisely
- equivalence relation
  - Congruent w.r.t. any context in first order
  - Not congruent w.r.t. input (and higher-order) context